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M.Sc. (Sem. - II) Examination September - 2025

Physics (PH - 421: Quantum Mechanics-I)

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) Symbols used have their usual meaning. (3) Figures to the right indicate full marks.	Student's Signature

Q. 1 Attempt any two questions out of the *(a), (b)* and *(c)* below.

(a) (i) Establish a relation between position and momentum representations. **5**

(ii) Consider the states $|\psi\rangle = 3i|\phi_1\rangle - 7i|\phi_2\rangle$ and $|\chi\rangle = -|\phi_1\rangle + 2i|\phi_2\rangle$, where $|\phi_1\rangle$ and $|\phi_2\rangle$ are orthonormal. **4**

Calculate:

(1) $|\psi + \chi\rangle$ and $\langle\psi + \chi|$

(2) $\langle\psi|\chi\rangle$ and $\langle\chi|\psi\rangle$.

(b) (i) Show that, if two Hermitian operators, $\hat{A}$ and $\hat{B}$, commute and if $\hat{A}$ has no degenerate eigenvalue, then each eigenvector of $\hat{A}$ is also an eigenvector of $\hat{B}$, and can construct a common orthonormal basis that is made of the joint eigenvectors of $\hat{A}$ and $\hat{B}$. **5**

(ii) Show that if $\hat{A}^{-1}$ exists, the eigenvalues of $\hat{A}^{-1}$ are just the inverse of those of $\hat{A}$. **4**

(c) (i) Define parity operator. Discuss its properties in detail. **5**

(ii) Show that: **4**

(1) The commutator of two Hermitian operators is anti-Hermitian.

(2) $\left[\hat{A},\left[\hat{B},\hat{C}\right]\right] + \left[\hat{B},\left[\hat{C},\hat{A}\right]\right] + \left[\hat{C},\left[\hat{A},\hat{B}\right]\right] = 0.$

Q. 2 Attempt any two questions out of the *(a), (b)* and *(c)* below.

(a) (i) Derive time-independent Schrodinger equation and its solution for a particle of mass m moving in a time-independent potential $\hat{V}(\vec{r})$. Why is this solution called stationary state? **5**

(ii) A particle of mass m, which moves freely inside an infinite potential well of length a, has the following initial wave function at $t = 0$: **4**

$$\psi(x, 0) = \frac{A}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}} \sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}} \sin\left(\frac{5\pi x}{a}\right)$$

where A is a real constant. Find A so that $\psi(x, 0)$ is normalized.

(b) (i) Prove that if two observables are compatible, their corresponding operators possess a set of common eigenstates. **5**

(ii) Consider a system whose state is given in terms of an orthonormal set of three vectors: $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$ as: $|\psi\rangle = \frac{\sqrt{3}}{3}|\phi_1\rangle + \frac{2}{3}|\phi_2\rangle + \frac{\sqrt{2}}{3}|\phi_3\rangle$. **4**

Considering an ensemble of 810 identical systems, each one of them in the state $|\psi\rangle$. If measurements are done on all of them, how many systems will be found in each of the states $|\phi_1\rangle$, $|\phi_2\rangle$ and $|\phi_3\rangle$?

(c) (i) Prove that probability is a conserved quantity by deriving the expressions of probability and current densities. **5**

(ii) What are the mathematical requirements that a wave function must satisfy to represent a physical system? Which among the following functions represent physically acceptable wave functions? **4**
$f(x) = 6 \sin \pi x$, $g(x) = 5 - |x|$, $h(x) = 9x$, $e(x) = x^2$

Q. 3 **Attempt any two questions out of the *(a), (b)* and *(c)* below.**

(a) (i) Solve time independent Schrodinger equation in one dimension for potential step in the case of $E > V_0$. **5**

(ii) Calculate the zero point energy for a particle in an infinite potential well for an electron confined to a 10^{-10} m atom. **4**

(b) (i) Solve time independent Schrodinger equation in one dimension for infinite asymmetric square well potential. **5**

(ii) A proton is subject to a harmonic oscillator potential $V(x) = \frac{m\omega^2 x^2}{2}$, $\omega = 5.34 \times 10^{21}\, S^{-1}$. Find the exact energies in MeV of the five lowest states. **4**

(c) (i) In the case of Harmonic oscillator, show that $\hat{a}$ is a annihilation operator and $\hat{a}\,|\,n\rangle\,\hat{a} = \sqrt{n}\,|\,n-1\rangle$. **5**

(ii) Show that $\hat{N} = \hat{a}\hat{a}^\dagger$ is occupation number operator and Hermitian. **4**

Q. 4 **Attempt any two questions out of the *(a), (b)* and *(c)* below.**

(a) (i) Find the matrices representing the operators $\hat{j}_+$ and $\hat{j}_z$ for $j = 1$. **4**

(ii) Calculate the commutators: (1) $[P_x, L_z]$, (2) $[L_z, L_y]$ **4**

(b) (i) For Pauli matrices, show that $\sigma_j^2 = \hat{I}\,(j = x,y,z)$. **4**

(ii) Show that: (1) $[\hat{j}_Z, \hat{j}_\pm] = \pm\, \hbar \hat{j}_\pm$, (2) $[\hat{j}_+, \hat{j}_-] = 2\,\hbar \hat{j}_z$. **4**

(c) (i) Derive the matrices for S_x, S_y and S_z for a particle with spin ½. **4**

(ii) Discuss the experimental evidence of spin. **4**